

Calcul \bar{A} (la matrice adjointe.)

$$A' = \begin{pmatrix} 4 & 3 & 1 \\ -5 & 2 & 1 \\ 0 & -1 & 1 \end{pmatrix} \text{ (0,25)}$$

$$\begin{pmatrix} -(2+1) - (-5+1) + (5+0) \\ -(3+1) + (4+0) - (4+0) \\ +(3+2) - (4+1) + (3+1) \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} + \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} & - \begin{vmatrix} -5 & 1 \\ 0 & 1 \end{vmatrix} & + \begin{vmatrix} -5 & 2 \\ 0 & -1 \end{vmatrix} \\ - \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} & + \begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} & - \begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} \\ + \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} & - \begin{vmatrix} 4 & 1 \\ -5 & 1 \end{vmatrix} & + \begin{vmatrix} 4 & 3 \\ -5 & 2 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 3 & 5 & 5 \\ -4 & 4 & 4 \\ 1 & -9 & 23 \end{pmatrix} \text{ (0,25)}$$

③ Calcul A^{-1} ; $A^{-1} = \begin{pmatrix} \frac{3}{32} & \frac{5}{32} & \frac{5}{32} \\ -\frac{4}{32} & \frac{4}{32} & \frac{4}{32} \\ \frac{1}{32} & -\frac{9}{32} & \frac{23}{32} \end{pmatrix} \text{ (0,30)}$

1] $A \cdot X = B \Leftrightarrow \begin{pmatrix} 4 & -5 & 0 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix}$

$$\Leftrightarrow \begin{cases} 4x - 5y = -4 \\ 3x + 2y - z = -5 \\ x + y + z = 1 \end{cases} \text{ (0,5)}$$

2] La solution unique du SEL: $X = A^{-1} \cdot B$

$$\begin{pmatrix} \frac{3}{32} & \frac{5}{32} & \frac{5}{32} \\ -\frac{4}{32} & \frac{4}{32} & \frac{4}{32} \\ \frac{1}{32} & -\frac{9}{32} & \frac{23}{32} \end{pmatrix} \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ (0,2)}$$

$$x = \begin{pmatrix} \frac{3}{32} & \frac{5}{32} & \frac{5}{32} \end{pmatrix} \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix} = \frac{-12}{32} - \frac{25}{32} + \frac{5}{32} = \frac{-72 - 25 + 5}{32} = \frac{-32}{32} = \boxed{-1} \text{ (0,4)}$$

$$y = \begin{pmatrix} -\frac{4}{32} & \frac{4}{32} & \frac{4}{32} \end{pmatrix} \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix} = \frac{-4}{32}(-4) + \frac{4}{32}(-5) + \frac{4}{32}(1) = \frac{+16 - 20 + 4}{32} = \boxed{0} \text{ (0,2)}$$

$$z = \begin{pmatrix} \frac{1}{32} & -\frac{9}{32} & \frac{23}{32} \end{pmatrix} \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix} = \frac{-4}{32} + \frac{45}{32} + \frac{23}{32} = \frac{64}{32} = \boxed{2} \text{ (0,2)}$$

$$X = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$